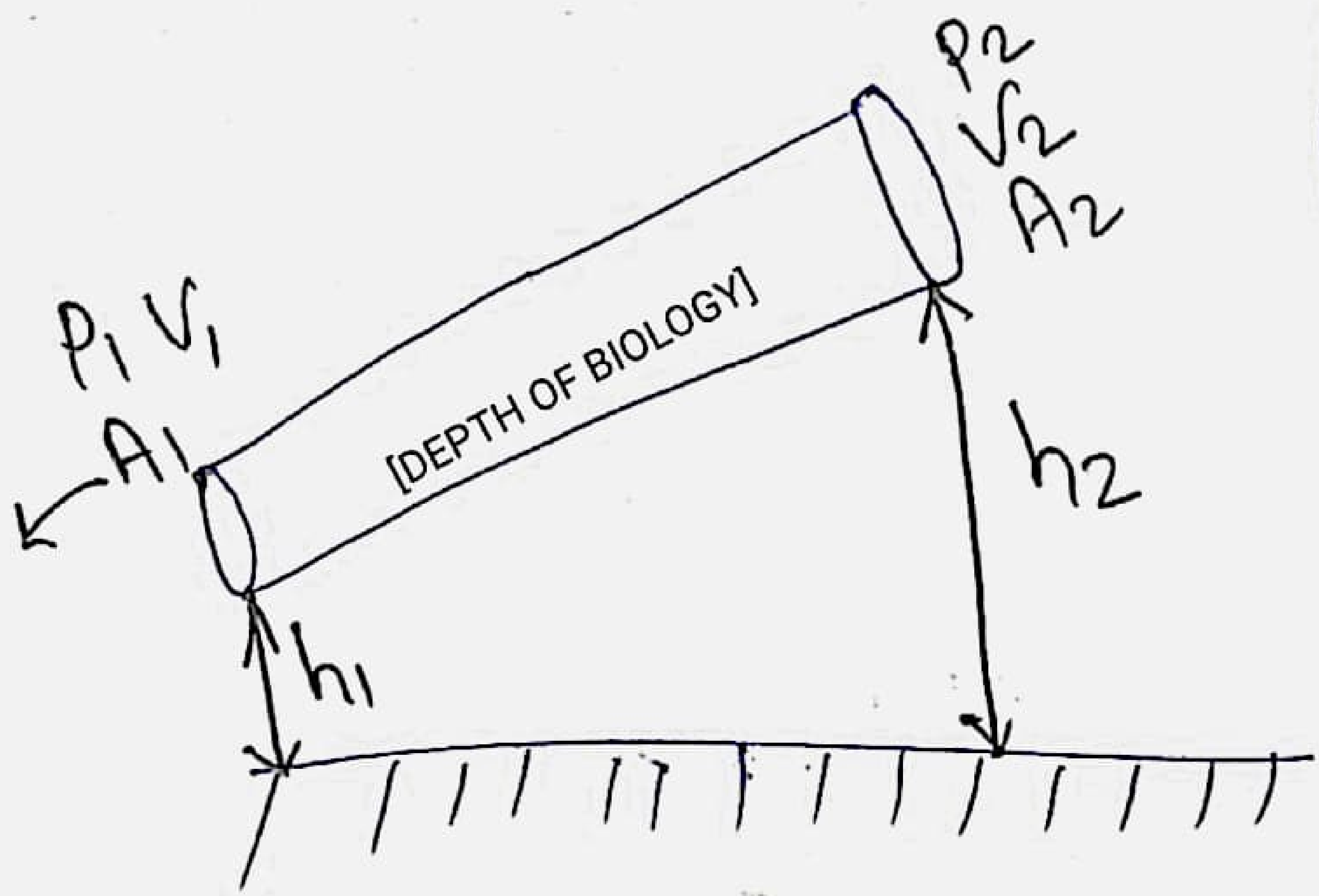
Bernoulli's Equation

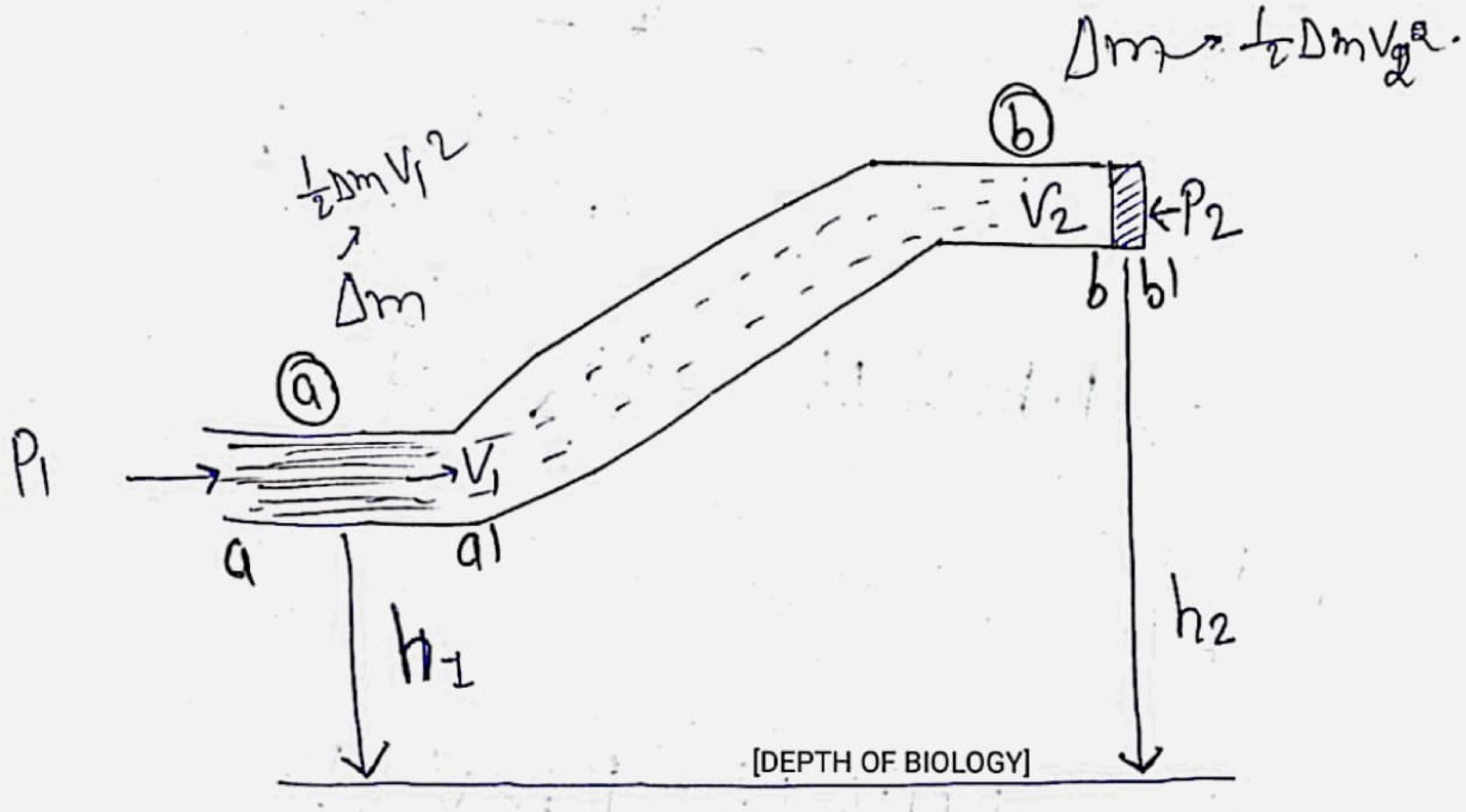
The sum of Pressure energy + Kinetic energy & Potential energy of a Ideal fluid flowing along a Streamline is a Constant.

Ideal fluid → Non-Viscous. → Steady flow.

$$\left[\underbrace{P}_{\text{Pressure Energy}} + \underbrace{\frac{1}{2}\rho v^2}_{\text{Kinetic energy}} + \underbrace{\rho g h}_{\text{Potential energy}} = \text{Constant}\right] \rightarrow \text{per Unit Volume.}$$

(Conservation of Energy)





We are Considering fluid Umber (a-b).

After time (t)

$a\text{-}b \longrightarrow (a'\text{-}b')$

$\Rightarrow$ Work Energy Theorem.

Work done $\leftarrow$ [Wall forces = Δ K.E]

① Work done by fluid Pressure.

$$W_{fluid\,press.} = W_{aa'} \ominus W_{bb'}$$

bcz. P_2 apply but flow of fluid in P_1.

$$\Rightarrow P_1 A_1 V_1 \Delta t - P_2 A_2 V_2 \Delta t$$

Work done by P_1.

[DEPTH OF BIOLOGY]

$$W_{fluid\,press.} = P_1 A_1 V_1 \Delta t - P_2 A_2 V_2 \Delta t$$

$$\begin{bmatrix} \Delta m = \rho \times Volume \\ \Delta m = \rho \times A_1 V_1 \Delta t \\ \Delta m = \rho \times A_2 V_2 \Delta t \end{bmatrix}$$

$d = \frac{m}{V}$

$\frac{\Delta m}{\rho} = Vol.$

$$\left[W_{f.p} = P_1 \frac{\Delta m}{\rho} - P_2 \frac{\Delta m}{\rho} \right]$$

[DEPTH OF BIOLOGY]

$$W_{fluid\,Pressure} \Rightarrow \boxed{\frac{\Delta m}{\rho} (P_1 - P_2)} \quad ①$$

(ii) Work done by Gravity →

$$\left[\underbrace{W_c}_{\text{Conservative change}} = -\underbrace{\Delta U}_{\text{change in Potential energy}} \right]$$

$$\left(W_c = U_{i_{\text{nitial}}} - U_{F_{\text{inal}}} \right)$$

$$= U(a-b) - U(a'-b')$$

$$= U_{aa'} + \cancel{U_{a'b}} - \cancel{U_{a'b}} - U_{bb'}$$

$$W_c = U_{aa'} - U_{bb'}$$

(2) $$\boxed{W_c = \Delta m g h_1 - \Delta m g h_2}$$

[DEPTH OF BIOLOGY]

$$W_{\text{all forces}} = \Delta KE.$$

$$\frac{\Delta m}{\rho}(P_1 - P_2) + \Delta m g h_1 - \Delta m g h_2 = K_F - K_i.$$

$$= K_{a'b'} - K_{ab}$$

$$= K_{a'b'} + K_{bb'} - (K_{aa'} + K_{a'b})$$

$$= \cancel{K_{a'b}} + K_{bb'} - K_{aa'} - \cancel{K_{a'b}}$$

$$\frac{\Delta m}{\rho}(P_1-P_2) + \Delta m g h_1 - \Delta m g h_2 = K_{bb'} - K_{aa'}$$

$$\frac{\cancel{\Delta m}}{\rho}(P_1-P_2) + \cancel{\Delta m} g h_1 - \cancel{\Delta m} g h_2 = \frac{1}{2}\cancel{\Delta m} v_2^2 - \frac{1}{2}\cancel{\Delta m} v_1^2$$

से multiply

$$\boxed{\frac{1}{\cancel{\rho}}\cancel{\rho}(P_1-P_2)\ \rho g h_1 - \rho g h_2 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2}$$

$$\boxed{P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2}$$

$\downarrow$

Bernoullis Equation

Hence Proved